

Digital

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الاستاذ

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ماضرة [4]

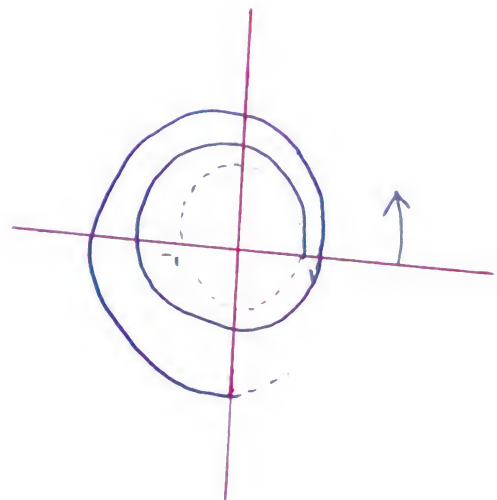
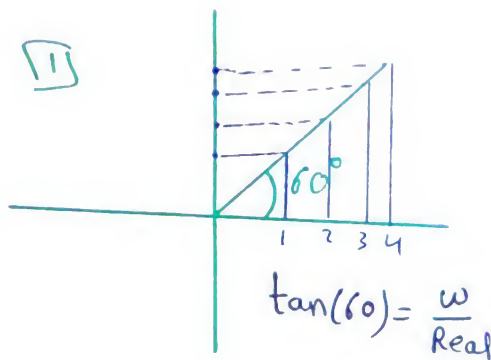
Remember:

$$S = \sigma + j\omega \Rightarrow z = e^{TS} = \begin{matrix} | & | & \angle \\ = & r & \angle \theta \end{matrix}$$

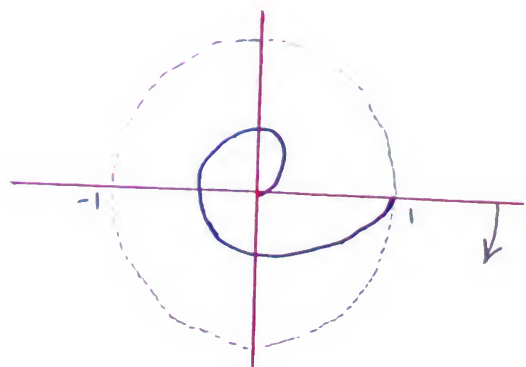
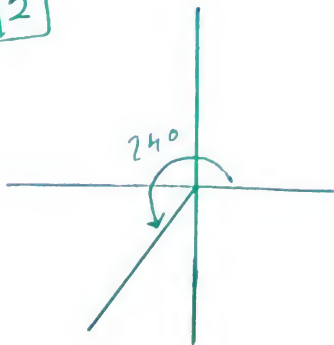
$$| | = r = e^{\sigma T} \quad \& \quad \theta = \omega T$$

Quiz Solution

[1]



[2]



Review Mapping From Digital. Lec. 03. pdf

Pages: 11, 12, 13

check system stability :

1] poles location :-

ex1: ch. eqn

$$z^2 - z + 0.5 = 0$$

$$\text{poles} \Rightarrow z_{1,2} = 0.5 \pm j0.5$$

$$|z_{1,2}| = \sqrt{(0.5)^2 + (0.5)^2} = 0.707 < 1 \quad [\text{Stable}]$$

ex2: ch. eqn

$$(z - 0.5)(z + 0.7)(z - 1) = 0$$

$$\text{poles} \Rightarrow z_{1,2,3} = 0.5, -0.7, 1$$

$$|z_{1,2,3}| = 0.5, 0.7, 1$$

↓ ↓ ↓
<1 <1 =1
Critically Stable

ex3: ch. eqn

$$(z - 0.5)(z + 0.7)(z - 1)(z + 1.5) = 0$$

$$\text{poles} \Rightarrow z_1 = 0.5, z_2 = -0.7, z_3 = 1, z_4 = -1.5$$

$$|z_{1,2,3,4}| = 0.5, 0.7, 1, 1.5$$

↑ ↑ ↑ ←
stable critical unstable

∴ the system is unstable

⇒ Continue

[2] Using bilinear transformation

$$z = e^{Ts} = e^{T/2 s} \cdot e^{T/2 s}$$
$$= \frac{e^{T/2 s}}{e^{-T/2 s}}$$

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$$

* For small values of $x \Rightarrow e^x \approx 1 + x$

$$z = \frac{1 + \frac{T}{2} s}{1 - \frac{T}{2} s}$$

* assume $\frac{T}{2} s = r$

$$z = \frac{1+r}{1-r} \quad (\text{stability can be checked using routh method})$$

Ex: ch. equation

$$z^3 + 3.3z^2 + 3z + 0.8 = 0$$

Check system stability using bilinear transformation

$$z = \frac{1+r}{1-r}$$

$$\left(\frac{1+r}{1-r}\right)^3 + 3.3\left(\frac{1+r}{1-r}\right)^2 + 3\left(\frac{1+r}{1-r}\right) + 0.8 = 0$$

Multiply by $(1-r)^3$

$$(1+r)^3 + 3.3(1+r)(1-r) + (1+r)(1-r)^2 + 0.8(1-r)^3 = 0$$

$$(1+r)(1+2r+r^2) + 3.3(1+2r+r^2)(1-r) + 3(1+r)(1-2r+r^2) + 0.8(1-r)(1-2r+1) = 0$$

$$\Rightarrow \boxed{r^3 + 9r^2 - 9r - 8 = 0} \quad (\text{ch. eqn in } r\text{-domain})$$

Using Routh method

r^3	1	-9
r^2	9	-81
r^1	0	
r^1	18	
r^0	-81	

$$A(r) = 9r^2 - 81$$

$$\frac{dA(r)}{dr} = 18r$$

UNstable since there is a change in sign

Using Calculator $z_{1,2,3} = -2, -0.5, -0.8$

$$|z_{1,2,3}| = 2, 0.5, 0.8$$

↳ unstable

$$\text{Ex2: } z^3 - 0.2z^2 - 0.25z + 0.05 = 0$$

$$z = \frac{1+r}{1-r}$$

ch. eqn \Rightarrow

$$\left(\frac{1+r}{1-r}\right)^3 - 0.2\left(\frac{1+r}{1-r}\right)^2 - 0.25\left(\frac{1+r}{1-r}\right) + 0.05 = 0$$

$$(1+r)^3 - 0.2(1+r)^2(1-r) - 0.25(1+r)(1-r)^2 + 0.05(1-r)^3 = 0$$

$$0.9r^2 + 3.6r^2 + 2.9r + 0.6 = 0$$

r^3	0.9	$\rightarrow 2.9$
r^2	3.6	$\rightarrow 0.6$
r^1	2.75	
r^0	0.6	

there is no sign changes
in first column \Rightarrow stable system

\Rightarrow Continue

3] Using Jury test

* assume that the ch. equation $F(z) = 1 + \overline{GH}(z) = 0$
 * in general form ch. eqn

$$1 + \overline{GH}(z) = F(z) = a_n z^n + a_{n-1} z^{n-1} + a_{n-2} z^{n-2} + \dots + a_1 z + a_0 = 0$$

* for the system to be stable:

① $F(1) > 0$

② $(-1)^n F(-1) > 0$

$n \rightarrow$ system order

③ $|a_0| < |a_n|$

} Sufficient for
2nd order systems

— For $n > 2$

Construct Jury Array

row no.	z^0	z^1	z^2	...	z^{n-1}	z^n
1	a_0	a_1	a_2	...	a_{n-1}	a_n
2	a_n	a_{n-1}	a_{n-2}	...	a_1	a_0
3	b_0	b_1	b_2	...	b_{n-1}	
4	b_{n-1}	b_{n-2}	b_{n-3}	...	b_0	
...						
$2n-3$	r_0	r_1	r_2	...		

عدهای حاصله
بجای ①

تکرار ①

تکرار ①

تکرار ①

→ continue

$$b_0 = \begin{vmatrix} a_0 & a_n \\ a_n & a_0 \end{vmatrix} ; b_1 = \begin{vmatrix} a_0 & a_{n-1} \\ a_n & a_1 \end{vmatrix}$$

$$b_2 = \begin{vmatrix} a_0 & a_{n-2} \\ a_n & a_2 \end{vmatrix}$$

Annotations:

- For b_0 : a_0 is labeled "الحد الثاني" (second term) with a downward arrow, and a_n is labeled "عند الحد الثاني" (at the second term) with an upward arrow.
- For b_1 : a_n is labeled "الحد الثاني من اليسار" (second term from the left) with a downward arrow, and a_1 is labeled "الحد الثاني من اليمين" (second term from the right) with a downward arrow.

④ $|b_0| > |b_{n-1}|$ القيمة الأولى والأخيرة

⑤ $|r_0| > |r_2|$

Ex: - ch. eqn : $F(z) = z^3 - 1.8z^2 + 1.05z - 0.2 = 0$

① $F(1) > 0$?

$= 1 - 1.8 + 1.05 - 0.2 = 0.05 > 0 \checkmark$

② $(-1)^{n=3} F(-1) > 0$?

$\Rightarrow -(-1 - 1.8 - 1.05 - 0.2) > 0 \checkmark$

③ $|a_0| < |a_3|$?

$0.2 < 1 \checkmark$

	z^0	z^1	z^2	z^3
1	-0.2	1.05	-1.8	1
2	1	-1.8	-1.05	-0.2
3	b_0	b_1	b_2	تكون عناصر فقط

$$b_0 = \begin{vmatrix} -0.2 & 1 \\ 1 & -0.2 \end{vmatrix} = -0.96$$

$$b_1 = \begin{vmatrix} -0.2 & -1.8 \\ 1 & 1.05 \end{vmatrix} = -1.59 \leftarrow \text{مطلوب}$$

$$b_2 = \begin{vmatrix} -0.2 & 1.05 \\ 1 & -1.8 \end{vmatrix} = -0.69$$

④ $|b_0| > |b_2|$?

$$0.96 > 0.69 \quad \checkmark$$

the system is stable

Ex2: ch. equation

$$F(z) = z^4 - 1.7z^3 + 1.04z^2 - 0.268z + 0.024 = 0$$

① $F(1) > 0$? \checkmark
0.096

② $(-1)^n F(-1) > 0$? \checkmark
 $F(-1) > 0 \quad \checkmark \checkmark$

③ $|a_0| = 0.024 < |a_4| = 1 \quad \checkmark$

	z^0	z^1	z^2	z^3	z^4
1	0.024	-0.268	1.04	-1.7	1
2	1	-1.7	1.04	-0.268	0.024
3	b_0	b_1	b_2	b_3	
4	b_3	b_2	b_1	b_0	
5	c_0	c_1	c_2		

\Rightarrow continue

EX2: Ch. eqn

$$b_0 = \begin{vmatrix} 0.024 & 1 \\ 1 & 0.024 \end{vmatrix} = -0.999 \approx -1$$

$$b_1 = \begin{vmatrix} 0.024 & -1.7 \\ 1 & -0.268 \end{vmatrix} = 1.693$$

$$b_2 = \begin{vmatrix} 0.024 & 1.04 \\ 1 & 1.04 \end{vmatrix} = 1.015$$

$$b_3 = \begin{vmatrix} 0.024 & -0.268 \\ 1 & -1.7 \end{vmatrix} = 0.227$$

$$\boxed{4} \quad |b_0| = 1 > |b_3| = 0.227 ? \checkmark$$

$$c_0 = \begin{vmatrix} b_0 = -1 & 0.227 \\ b_3 = 0.227 & -1 \end{vmatrix} = 0.943$$

$$c_2 = \begin{vmatrix} -1 & 1.693 \\ 0.227 & -1.015 \end{vmatrix} = 0.63$$

$$\boxed{5} \quad |c_0| = 0.943 > |c_2| = 0.63 ? \checkmark$$

System is stable

\Rightarrow continue

Ex 3: ch. equation

$$F(z) = z^3 + 3.3z^2 + 3z + 0.8 = 0$$

1) $F(1) > 0?$ ✓

2) $(-1)^{n=3} F(-1) > 0?$

$-(-1 + 3.3 - 3 + 0.8) > 0?$ X

System is unstable